BRIEF COMMUNICATIONS

THE PROBLEM OF THE FREE CONVECTION NEAR A HORIZONTAL CYLINDER WITH CONSTANT THERMAL FLUX AT THE SURFACE

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UDC 536.25

Using Shvets's method we solve the problem of the stationary free convection of a viscous fluid near a horizontal cylinder with constant thermal flux at the surface. From the solution we find the thickness of the laminar boundary layer at the cylinder and the temperature and velocity distributions in it.

The problem of the stationary free thermal convection near a heated horizontal cylinder of infinite length in a viscous fluid was considered in [1, 2] for the case when the heat flux is specified at the surface of the cylinder. In [1] the solution was reduced to the numerical integration of the ordinary differential equations obtained from the equations of free convection in the boundary layer approximation by expanding the stream function and the temperature in series in powers of x. In the second paper the problem was solved by replacing the differential equations of free convection by integral equations and then representing the velocity and temperature distributions in the boundary layer as polynomials with three terms in powers of y/δ, and numerically integrating one of the resulting equations.

Below we give the solution of the problem by the method of successive approximations in analytic form for the case of constant thermal flux. We assume that near the cylinder there is a laminar boundary layer.

The equations for the conservation of momentum, energy, and mass for a laminar boundary layer [3] near a horizontal cylinder in nondimensional form are:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\partial^2 u}{\partial y^2}+T\sin x, \tag{1}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{1}{\Pr}\cdot\frac{\partial^2 T}{\partial y^2}, \tag{2}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{3}$$

where

$$x=\frac{x_1}{R};\quad y=\frac{y_1}{R}(\mathrm{Gr}^*)^{1/5};$$

$$u=\frac{R}{\nu}u_1(\mathrm{Gr}^*)^{-2/5};\quad v=\frac{R}{\nu}v_1(\mathrm{Gr}^*)^{-1/5};$$

$$T=\frac{\lambda}{qR}(T_1-T_{1\infty})(\mathrm{Gr}^*)^{1/5}.$$

As the boundary conditions we take

$y=0$:

$$u=v=0,\quad \frac{\partial T}{\partial y}=-1, \tag{4}$$

Leningrad Institute of Mechanics. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 18, No. 4, pp. 727-730, April, 1970. Original article submitted June 18, 1969.

$y = \delta$:

$$u = 0, \qquad T = 0. \tag{5}$$

Here

$$\delta = \frac{\delta_1}{R} (\mathrm{Gr}^*)^{1/5}.$$

We shall solve Eqs. (1)-(3) with the boundary conditions (4), (5) using the approximate method of Shvets [4]. For the i-th approximation we have

$$\frac{\partial^2 T_i}{\partial y^2} = \mathrm{Pr}\left(u_{i-1}\frac{\partial T_{i-1}}{\partial x} - \frac{\partial T_{i-1}}{\partial y}\int_0^y \frac{\partial u_{i-1}}{\partial x}\,dy\right), \tag{6}$$

$$\frac{\partial^2 u_i}{\partial y^2} + T_i \sin x = u_{i-1}\frac{\partial u_{i-1}}{\partial x} - \frac{\partial u_{i-1}}{\partial y}\int_0^y \frac{\partial u_{i-1}}{\partial x}\,dy,$$

$$y = 0: \qquad u_i = 0, \qquad \frac{\partial T_i}{\partial y} = -1, \tag{7}$$

$$y = \delta_i: \qquad u_i = 0, \qquad T_i = 0.$$

To determine the thickness of the boundary layer in the i-th approximation we use the condition that there is no thermal flux at the outer boundary of the layer

$$\left.\frac{\partial T_i}{\partial y}\right|_{y=\delta_i} = 0. \tag{8}$$

For the zero-order approximation we take

$$u = 0, \qquad T = 0.$$

Then the equations of the first approximation are obtained in the form

$$\frac{\partial^2 T}{\partial y^2} = 0,$$

$$\frac{\partial^2 u}{\partial y^2} + T \sin x = 0.$$

Solving these simultaneously, with the above boundary conditions, we obtain equations for the temperature and velocity in the first approximation

$$T = \delta - y, \; u = \left(\frac{y^3}{6} - \frac{\delta y^2}{2} + \frac{\delta^2 y}{3}\right)\sin x. \tag{9}$$

Substituting these results again in the left sides of Eqs. (6) and (7), we obtain the second approximation equations. Solving these simultaneously, with the boundary conditions, we obtain equations for the temperature and velocity distributions in the boundary layer in the second approximation. For the temperature we have

$$T = \delta - y + \frac{1}{6}\mathrm{Pr}\left[\left(\frac{\delta y^3}{3} - \frac{y^4}{12} - \frac{\delta^4}{4}\right)\delta\cdot\frac{d\delta}{dx}\sin x + \left(\frac{y^6}{120} - \frac{\delta y^5}{20} + \frac{\delta^2 y^4}{12} - \frac{\delta^6}{24}\right)\cos x\right]. \tag{10}$$

Substituting the latter in (8) and carrying out the necessary operations, we obtain an equation for the thickness of the boundary layer

$$\delta^4 \frac{d\delta}{dx}\sin x + \frac{1}{5}\delta^5 \cos x = \frac{9}{\mathrm{Pr}},$$

which, using the substitution $z = \delta^5$ can be linearized:

$$\frac{dz}{dx}\sin x + z\cos x - \frac{45}{\mathrm{Pr}} = 0.$$

The solution of this equation, assuming that the thickness of the boundary layer is finite at the lower generator of the cylinder (x = 0) has the form

$$\delta = 2.14 \cdot \Pr^{-1/5}\left(\frac{x}{\sin x}\right)^{1/5}. \tag{11}$$

NOTATION

x_1	is the coordinate measured from the lower generator along the arc of the circumference of the cylinder cross section;
y_1	is the coordinate measured along the normal to the surface of the cylinder;
R	is the cylinder radius;
u_1	is the velocity component in the x_1-direction;
v_1	is the velocity component in the y_1-direction;
T_1	is the temperature;
$T_{1\infty}$	is the temperature of the undisturbed fluid;
q	is the specific thermal flux;
δ_1	is the thickness of the boundary layer;
$Gr^* = qg\beta R^4/\lambda\nu^2$	is the modified Grashof number;
$Pr = \nu/a$	is the Prandtl number;
ν	is the kinematic viscosity coefficient;
λ	is the thermal conductivity;
a	is the thermal diffusivity;
β	is the coefficient of volume expansion;
g	is the acceleration due to gravity.

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